A DE BROGLIE-LIKE WAVE IN THE PLANETARY SYSTEMS

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Abstract

In this work we do an "interpolation" of Scardigli theory of a quantum-like description of the planetary system that reproduces remarkable Titius-Bode-Richardson rule. More precisely, instead of simple, approximate, Bohr-like theory, or, accurate, Schrödinger-like theory, considered by Scardigli, we suggest originally a semi-accurate, de Broglie-like description of the planetary system. Especially, we shall propose a de Broglie-like waves in the planetary systems. More precisely, in distinction from Scardigly (which postulated absence of the interference phenomena at planet orbits) we shall prove that, roughly speaking, planets orbits equal a sum of natural numbers of two types, large and small, of the de-Broglie-like waves. It is similar to well-known situation in atomic physics by interpretation of Bohr momentum quantization postulate by de Broglie relation.

"Why you are miserable? - Because of my miseries
When Saturn packed my satchel I think
He put in these troubles - That's mad
You're his lord and you talk like his slave
Look what Solomon wrote in his book
"A wise man" he says, "has authority
Over the planets and their influence" -
I don't believe it, as they made me I'll be -
What are you saying? - Yes, that's I think -
I've nothing more to tell you - I'll survive without it-"

Francois Villon, "The Debate Between One and Other Villon Part,His Body And His Hart"

In this work we shall do an "interpolation" of Scardigli theory of a quantum-like description of the planetary system [1] that reproduces remarkable Titius-Bode-Richardson rule [2]. More precisely, instead of simple, approximate, Bohr-like (nave quantum-like) theory, or, accurate, Schrödinger-like (quantum-like) theory, considered by Scardigli, we shall suggest originally a semi-accurate, de Broglie-like (quasi-classical-like) description of the planetary system (without explicit Scardigli consideration). Especially, we shall propose a de Broglie-like waves in the planetary systems. More precisely, in distinction from Scardigly (which postulated absence of the interference phenomena at planet orbits) we shall prove that, roughly speaking, planets orbits equal a sum of natural numbers of two types, large and small, of de-Broglie-like waves. It is similar to well-known situation in atomic physics by interpretation of Bohr momentum quantization postulate by de Broglie relation (so that Bohr electron orbit holds natural number of corresponding de Broglie waves).

For a relatively small physical system with mass m, that stablely rotates, with linear speed v, along a circumference with radius R, about central, massive system M, for $M \gg m$, Scardigli [1] suggest the following rules

$$\frac{mv^2}{R} = G\frac{mM}{R^2} \tag{1}$$

$$\frac{J}{m} = vR = S\exp[\alpha n] \qquad \text{for} \qquad n = 1, 2, 3... \tag{2}$$

where $\frac{J}{m} = vR$ represents the angular momentum of small system over mass unit, while S and α represent some parameters independent of the natural number $n = 1, 2, 3,$. (Originally, i.e. in [1], Scardigli uses denotation λ instead of α. But, since in our work we shall consider wave length, usually denoted by λ, we change original Scardigli denotation by α. All other denotations in our work are identical to Scardigli denotations.)

As it is not hard to see, given rules are, in some degree conceptually similar to Bohr rules of electron motion within atom. However, Scardigli pointed out that given similarity is not the identity. He criticized some authors which use a non-modified Bohr quantization form, i.e. that in (2) instead of $\exp[\alpha n]$ use simply n, since it yields R as function of n^2 which contradicts to empirical facts. Scardigli observed that elimination of this contradiction, by conservation of non-modified Bohr quantization form (simple term n), cannot be realized physically plausiblely. Namely, it needs either non-consequitve natural numbers or an ad hoc separation of Solar system in the terrestrial and gigantic planets, any of which is physically non-plausible.

Simple solution of Scardigli equations (1), (2) are

$$v_n = \frac{GM}{S}\exp[-\alpha n] = v_1\exp[-\alpha(n-1)] \qquad \text{for} \qquad n = 1, 2, 3... \tag{3}$$

and

$$R_n = \frac{S^2}{GM}\exp[2\alpha n] = R_1\exp[2\alpha(n-1)] \qquad \text{for} \qquad n = 1, 2, 3... \tag{4}$$

where $R_1 = \frac{S^2}{GM}\exp[2\alpha]$ and $v_1 = \frac{GM}{S}\exp[-\alpha]$ represent first planet, i.e. Mercury radius and speed. Last term holds form identical to remarkable Titius-Bode rule in Richardson form for planet distances [2], under condition that $\frac{S^2}{GM}$ represents Titius-Bode-Richardson parameter a. It implies

$$S = (aGM)^{\frac{1}{2}} \tag{5}$$

so that, as it is necessary, S represents really a parameter independent of m. (It can be added that Richardson form, (4), (5), corresponds to usual form of Titius-Bode rule, only after neglecting of the second order corrections.)

Further, Scardigli [1] suggest a more accurate, Schrődinger-like theory of the planetary systems, which, as its simplification, holds mentioned, simple, Bohr-like theory.

Now, we shall consider originally a semi-accurate, quasi-classical, de Broglie-like description of the planetary systems (without explicit Scardigli consideration), that can be considered as an "interpolation", between Bohr-like and Schrődinger-like theory. Especially, we shall propose a de Broglie-like waves in the planetary systems (explicitly suggested by Scardigli).

As it is well-known, de Broglie wave, corresponding to a quantum particle with mass m and speed v, holds wave length $\lambda = \frac{h}{mv}$, where h represents Planck constant. It is well-known too that Bohr momentum quantization rule, ad hoc postulated within particle concept of quantum system, $mv_n r_n = n\frac{h}{2\pi}$ for $n = 1, 2,$, can be simply interpreted by given de Broglie relation. Namely, given relation simply yields $2\pi r_n = n\frac{h}{mv_n} = n\lambda_n$ for $n = 1, 2,$. It means that any electron orbit in Bohr atom holds natural (positive integer) number of corresponding de Broglie wave lengths. In other words, Bohr quantization rule can be considered as a stability (equilibrium) condition for de Broglie waves corresponding to electron. Simply speaking, without Bohr orbits, electron de Broglie waves disappear, while, at Bohr orbits, electron de Broglie waves stand conserved by means of the interference effects (in a linear theory).

We shall suppose that in some degree similar situation exists implicitly in mentioned Scardigli theory. Namely, we shall suppose, according to (2), (3), (4),

$$v_n R_n = S \exp[\alpha n] = \frac{\sigma}{2\pi m} \exp[\alpha(n-1)] \qquad \text{for} \qquad n = 1, 2, 3... \tag{6}$$

where

$$\sigma = S(2\pi m) \exp[\alpha]. \tag{7}$$

It implies

$$2\pi R_n = \frac{\sigma}{mv_n} \exp[\alpha(n-1)] \qquad \text{for} \qquad n = 1, 2, 3.... \tag{8}$$

It, in conceptual similarity with de Broglie interpretation of Bohr momentum quantization rule, suggests that, according to (7) and (5), expression

$$\lambda_n = \frac{\sigma}{mv_n} = \frac{2\pi S \exp[\alpha]}{v_n} = \frac{2\pi (aGM)^{\frac{1}{2}} \exp[\alpha]}{v_n} \qquad \text{for} \qquad n = 1, 2, 3... \tag{9}$$

can be considered as de Broglie-like wave length of a planet. It is very interesting that given wave length, that depends of a and M, does not depend effectively of m, even if it is defined by means of m.

However, in distinction from Bohr momentum quantization postulate interpreted by de Broglie relation, expression

$$2\pi R_n = \lambda_n \exp[\alpha(n-1)] \qquad \text{for} \qquad n = 1, 2, 3... \tag{10}$$

, obtained by (8) and (9), cannot be, seemingly, simply interpreted as a stability (equilibrium) condition (realized by means of interference effects) within wave dynamics. Meanwhile, (10) can be approximated by first order Taylor expansion of $\exp[\alpha n]$ for n=1, 2, 3, , which yields

$$2\pi R_n = \lambda_n (1 + \alpha(n-1)) = \lambda_n + (n-1)\lambda_{nred} \qquad \text{for} \qquad n = 1, 2, 3... \tag{11}$$

where

$$\lambda_{nred} = \lambda_n \alpha \qquad \text{for} \qquad n = 1, 2, ... \tag{12}$$

The last term, i.e. λ_{nred}, can be considered as an effective reduction, or, simply, small de Broglie-like wave length corresponding to large de Broglie-like wave length, λ_n, for $n = 1, 2, 3,$. In other words, it can be supposed that here is a non-linear wave dynamics whose stability (equilibrium) condition (11) includes simultaneously two types of the waves. (It would correspond to frequency mixing and similar phenomena in non-linear optics too.) First one is usual, i.e. large de Broglie-like wave, with de Broglie-like wave length λ_n, for $n = 1, 2, 3,$ (9). Second one is corresponding, reduced, i.e. small de Broglie-like wave with reduced, i.e. small de Broglie-like wave length λ_{nred}, for $n = 1, 2, 3,$ (12). Then condition (11) simply means that circumference of n-th planetary orbit holds only one large de Broglie-like wave length λ_n and $n-1$ corresponding reduced, i.e. small de Broglie-like wave lengths λ_{nred} for $n = 1, 2, 3,$. It is in a satisfactory conceptual similarity with Bohr momentum quantization postulate interpreted by de Broglie relation. Especially, as it is not hard to see, for the first planet (11) has form (without small wave length) completely analogous to Bohr quantization postulate for the first electron orbit.

Further, we shall consider two especial approximation limits of (11).

Firstly, (11) turns out approximately in

$$2\pi R_n \simeq \lambda_n \quad \text{for} \quad 1 \geq \alpha(n-1) \quad \text{or} \quad n \leq \frac{1}{\alpha} + 1. \tag{13}$$

It means that here circumference of n-th planetary orbit is approximately equivalent to large de Broglie-like wave length λ_n, for $n \leq \frac{1}{\alpha}$. Meanwhile, (13) has form that is not completely analogous to form of de Broglie interpretation of Bohr momentum quantization postulate (there is no n before λ_n at right hand of (13)).

Secondly, (11) turns out approximately in

$$2\pi R_n \simeq (n-1)\lambda_{nred} \quad \text{for} 1 \quad < \alpha(n-1) \quad \text{or} \quad \text{n} > \frac{1}{\alpha} + 1. \tag{14}$$

It means that here circumference of n-th planetary orbit is approximately equivalent to $n-1$ small de Broglie-like wave length λ_n, for $n > \frac{1}{\alpha} + 1$. Obviously, (14) has form almost completely analogous to form of de Broglie interpretation of Bohr momentum quantization postulate. Namely here n-th planet orbit holds approximately $n-1$ corresponding reduced de Broglie-like wave lengths while n-th electron orbit holds n de Broglie wave length.

It can be observed that in Solar system $\alpha = 0.2685$ that implies $\frac{1}{\alpha} = 3.7239 + 15$. It practically means that condition (13) is satisfied for terrestrial planets and asteroids belt, while condition (14) is satisfied for gigantic planets. Thus, we obtain separation of Solar system in terrestrial (including asteroids belt) and gigantic planets in a reasonable way, i.e. by two opposite limits of (11).

In conclusion we shall discuss obtained results. It can be supposed [1], [2] (and references therein), that after formation of Sun as massive central body rest of Sun system can be satisfactorily treated as a real fluid-like system (consisting really of the dust, gas etc.). Its non-linear dynamics can be, in principle, accurately and completely described by some, non-linear, e.g. modified, Schrödinger-like equation or similar partial differential equation. Solution of given equation can accurately and completely predict how fluid density evolves during time forming finally a discrete series of the splited parts corresponding to permitted orbits of the future planets. (A

planet can be considered as the "condensate" of given fluid at permitted orbits.) However, formulation as well as solution of given equation is very complex and can be connected with many technical as well as principal problems. For this reason an approximation of given dynamics can be made by suggested de Broglie-like theory. Instead of a complete dynamical evolution of the Sun system fluid only final stable planets orbits can be proposed by corresponding to Bohr-like, i.e. Scardigli, momentum quantization postulate. It can be interpreted de Broglie-like waves (and interference rules) as a stability condition in non-linear wave dynamics (with mixing frequency, i.e. wave lengths effects). Roughly speaking future planets orbits correspond to domains that hold the sum of one large de Broglie-like wave length and integer number of small de Broglie-like waves. In given domains interference effects yields maximal amplification of the fluid density. In further approximation orbits of the future terrestrial planets correspond to domains that hold one large de Broglie-like wave length, on the one side. On the other side, within the same approximation, orbits of future gigantic planets correspond to domains that hold integer number of small de Broglie-like wave lengths. Last case is almost completely analogous to de Broglie interpretation of Bohr momentum quantization postulate. Without mentioned domains interference effects lead toward maximal decrease and final disappearance of the fluid density.

References

[1] F. Scardigli, *A Quantum-like Description of the Planetary Systems*, gr-qc/0507046, and references therein

[2] M. M. Nieto, *The Titius-Bode Law of Planetary Distances: its History and Theory* (Pergamon Press, Oxford, 1972)